

## MAT 111 - College Algebra

### Section 2.6: Combinations of Functions

#### Objectives:

1. Learn how to add, subtract, multiply and divide functions.
2. Learn how to find domain of the new functions formed by the above operations.
3. Learn how to compose functions and how to find the domain of the new function.
4. Learn how to "decompose" functions

Consider the functions

$$f(x) = 2x - 5$$

$$g(x) = 2 - x$$

Operation	Notation and Illustration
Addition	$\begin{aligned} (f + g)(x) &= f(x) + g(x) \\ &= (2x - 5) + (2 - x) \\ &= x - 3 \end{aligned}$
Subtraction ( $g$ from $f$ )	$\begin{aligned} (f - g)(x) &= f(x) - g(x) \\ &= (2x - 5) - (2 - x) \\ &= 3x - 7 \end{aligned}$
( $f$ from $g$ )	$(g - f)(x) = g(x) - f(x)$
Multiplication	$(fg)(x) = f(x)g(x)$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

**Examples:** Evaluate the following where  $f$  and  $g$  are as given above:

1.  $(f + g)(2) =$

2.  $(g - f)(-1) =$

3.  $(fg)(-1) =$

4.  $\left(\frac{f}{g}\right)(0) =$

The domain of an *arithmetic combination* of functions  $f$  and  $g$  consists of all real numbers that are common to the domains of  $f$  and  $g$ . In case of dividing two functions, we need to make sure to exclude values of  $x$  that make the denominator zero.

**Examples:** Find the domain of the following functions given  $f(x) = \sqrt{x^2 - 4}$  and  $g(x) = \sqrt{1 - x}$ .

1.  $f + g$
2.  $f/g$
3.  $g/f$

### Composition of Functions:

Two functions  $f$  and  $g$  can be composed in two ways:

1.  $f$  composed with  $g$  is written  $f \circ g$

$$(f \circ g)(x) = f(g(x))$$

**Example:**  $f(x) = x^2 - 4$  and  $g(x) = 1 - x$

$$(f \circ g)(x) =$$

2.  $g$  composed with  $f$  is written  $g \circ f$

$$(g \circ f)(x) = g(f(x))$$

**Example:**  $f(x) = x^2 - 4$  and  $g(x) = 1 - x$

$$(g \circ f)(x) =$$

**Example:** Find  $(f \circ g)(2)$  where  $f$  and  $g$  are as given above.

The domain of  $f \circ g$  is the set of all  $x$  in the domain of  $g$  such that  $g(x)$  is in the domain of  $f$ . The domain of  $g \circ f$  is the set of all  $x$  in the domain of  $f$  such that  $f(x)$  is in the domain of  $g$ .

**Example:** Find the domain of  $f \circ g$  and  $g \circ f$  where  $f(x) = \frac{3}{x^2 - 1}$  and  $g(x) = x + 1$ .

### Decomposition of Functions:

**Examples:** Find two functions  $f$  and  $g$  such that  $(f \circ g)(x) = h(x)$ .

1.  $h(x) = (1 - x)^3$

2.  $h(x) = \frac{4}{(5x + 2)^2}$