MAT 111 - College Algebra

Section 3.3 Polynomial Division

Objectives:

1. Learn how to divide a polynomial by another polynomial.
2. Learn the Remainder theorem.
3. Learn the Factor theorem.

1. There are two ways to divide a polynomial by another polynomial:

   (a) Using long division.

   (b) Using synthetic division (works when the divisor is in linear form).

   Examples:

   (a) Use long division to divide $4x^3 - 7x^2 - 11x + 5$ by $4x + 5$.

   (b) Find the quotient and remainder when $x^5 + 7$ is divided by $x^3 - 1$.

On dividing a polynomial function $f(x)$ by another polynomial function $d(x)$ we get a quotient $q(x)$ and a remainder $r(x)$ with $r(x) = 0$ or degree of r is strictly less than degree of d. The polynomial function $f(x)$ can then be written as

$$\begin{array}{ccccccc} f(x) & = & d(x) & q(x) & + & r(x) \\ \downarrow & & \downarrow & \downarrow & & \downarrow \\ \textit{Dividend} & = & \textit{Divisor} & \textit{Quotient} & + & \textit{Remainder} \end{array}$$

If $r(x) = 0$, then $d(x)$ divides the polynomial $f(x)$ and we have found a factor of the polynomial.

If the divisor, $d(x)$ is a linear function, then the degree of the remainder $r(x)$ is ....

Example: Divide $f(x) = x^3 - 5x^2 - 11x + 8$ by $x + 2$.

2. **If the divisor $d(x)$ is of the form $x - c$, then the remainder is ...**

   Example: Find the remainder on dividing $f(x) = x^3 - 4x^2 + 2x - 5$ by $x + 2$.

3. (a) **If $f(c) = 0$, then $x - c$ is a factor of $f(x)$.**

   (b) **If $x - c$ is a factor of $f(x)$, then $f(c) = 0$**

   Examples:

   (a) Determine whether $x - 2$ is a factor of $f(x) = x^3 - 7x + 6$ or not.

(b) Use long division to show that $x = \frac{2}{3}$ is a zero of $f(x) = 48x^3 - 80x^2 + 41x - 6$ and use the result to factor the polynomial completely. List all real zeros of the function.

(c) Given that $x + 3$ and $x - 2$ are two factors of $f(x) = 3x^3 + 2x^2 - 19x + 6$, find the remaining factor and list zeros of f.